Explicitly accounting for the heat sink strengths in the thermal matching of thermoelectric devices.
A unified practical approach

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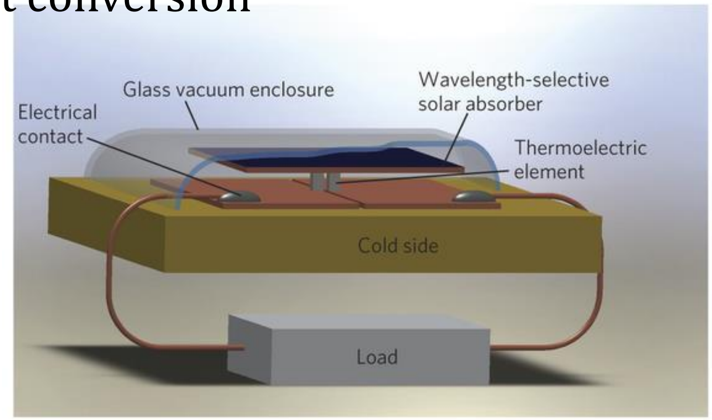
Summary

- Why accounting for heat sink strengths
- The 1D Model
- Dirichlet and Neumann solutions as special cases
- The general solution: impact of
  - dissipation
  - heat current
- Optimizing leg length in bulk and nanostructured systems
- Conclusions and outlook

Motivations

bulk heat conversion



al. et G. Chen, Nature Materials 10, 532–538 (2011)

Heat source strengths may widely differ, and this makes the choice of actual BCs critical

body heat harvesting

Sun Jin Kim et al., Energy Environ. Sci., 2014, 7, 1959-1965

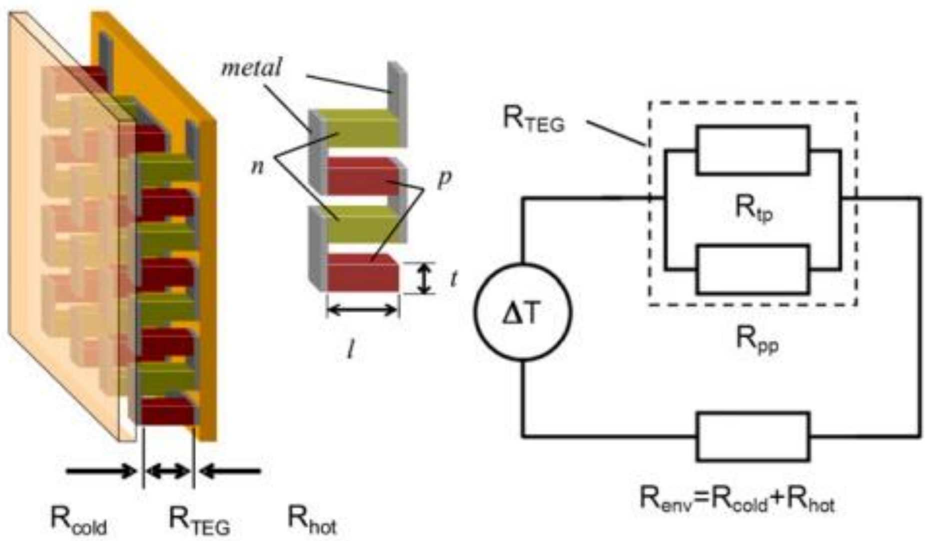
microharvesting

M. Codecasa et al., J. Electron. Mater., (2014) 10.1007/s11664-014-3297-9

The standard model

$$w = \frac{(\alpha \Delta T)^2}{R_{el}}$$

Standard models set as TE system the TEG + dissipators, solving heat (or Domenicali) equation by imposing either fixed-temperature or fixed-flow BCs.



In general, neither of them is correct:

- no real heat power sinks can fix boundary temperatures for arbitrarily small R_{th}
- no real heat power source can provide constant heat currents for extremely large R

V. Leonov, P. Fiorini, and R. J. Vullers, Microelectr. J. 42, 579 (2011)
V. Leonov and P. Fiorini, in Proc. 5th Eur. Conf. Thermoelectrics (2007), p. 129.

Geometry optimization in the standard model

As a result

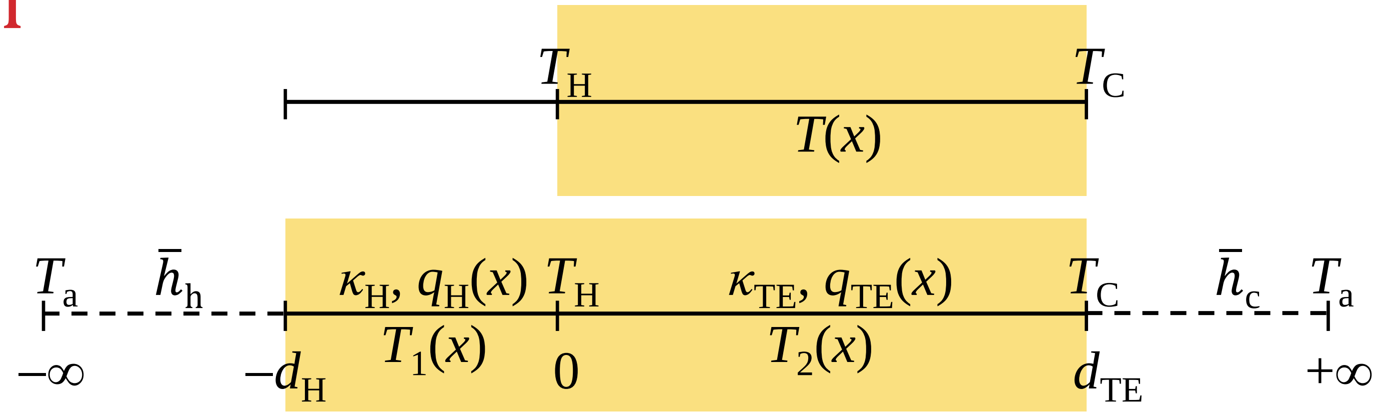
- under fixed-ϕ BCs for $R \to \infty$ both ΔT and $w \to \infty$ suggesting long TE legs and/or small TE cross sections

- under fixed-T BCs for $R \to 0$ both ϕ and $w \to \infty$ suggesting short TE legs and/or large TE cross sections

Optimization of geometry requires an *a priori* assumption about the BCs better approximating the actual scenario

Aim of this work is to propose a different way of modeling TEGs avoiding any BC stipulation

The model



In the proposed model the system encompasses the TEG, the heat source, and the heat sink. The whole system is in thermal contact with the ambient. Fixed-T (T_A) BC are there bound to apply.

Model equation

$$\begin{cases} \kappa_{\mathrm{TE}} T_2''(x) + q_{\mathrm{TE}}(x) = 0 \\ \bar{h}_{\mathrm{c}}(T_2(d_{\mathrm{TE}}) - T_{\mathrm{A}}) = -\kappa_{\mathrm{TE}} T_2'(d_{\mathrm{TE}}) \\ \kappa_{\mathrm{H}} T_1''(x) + q_{\mathrm{H}}(x) = 0 \\ \bar{h}_{\mathrm{h}}(T_1(-d_{\mathrm{H}}) - T_{\mathrm{A}}) = \kappa_{\mathrm{H}} T_1'(-d_{\mathrm{H}}) \\ \kappa_{\mathrm{TE}} T_2'(0) = \kappa_{\mathrm{H}} T_1'(0) \\ T_1(0) = T_2(0) \end{cases}$$

$$q_{\mathrm{H}}(x) \equiv \frac{\theta_{\mathrm{H}}}{d_{\mathrm{H}}} \Pi\left(\frac{x}{d_{\mathrm{H}}} + \frac{1}{2}\right) > 0$$

$$q_{\mathrm{TE}}(x) \equiv -\frac{\theta_{\mathrm{TE}}}{d_{\mathrm{TE}}} \Pi\left(\frac{x}{d_{\mathrm{TE}}} - \frac{1}{2}\right) < 0$$

Dimensionless equation

$$
\begin{cases}
v_2''(\hat{x}) = \mu_3 \Pi(\hat{x} - \tfrac{1}{2}) \\
v_2(1) = -\mu_5 v_2'(1) \\
v_1''(\hat{x}) = -\mu_2 \mu_1^{-2} \Pi\left(\hat{x}\mu_1^{-1} + \tfrac{1}{2}\right) \\
v_1(-\mu_1) = \mu_4 \mu_1 v'_1(-\mu_1) \\
v_1'(0) = \mu_6 v_2'(0) \\
v_1(0) = v_2(0)
\end{cases}
$$

Reduced variables:

$$\hat{x} \equiv x / d_{\mathrm{TE}}$$

$$v(\hat{x}) \equiv T(x) / T_A - 1$$

$$\mu_1 \equiv d_{\mathrm{H}} / d_{\mathrm{TE}}$$

$$\mu_2 \equiv \theta_{\mathrm{H}} d_{\mathrm{H}} / (T_{\mathrm{A}} \kappa_{\mathrm{H}})$$

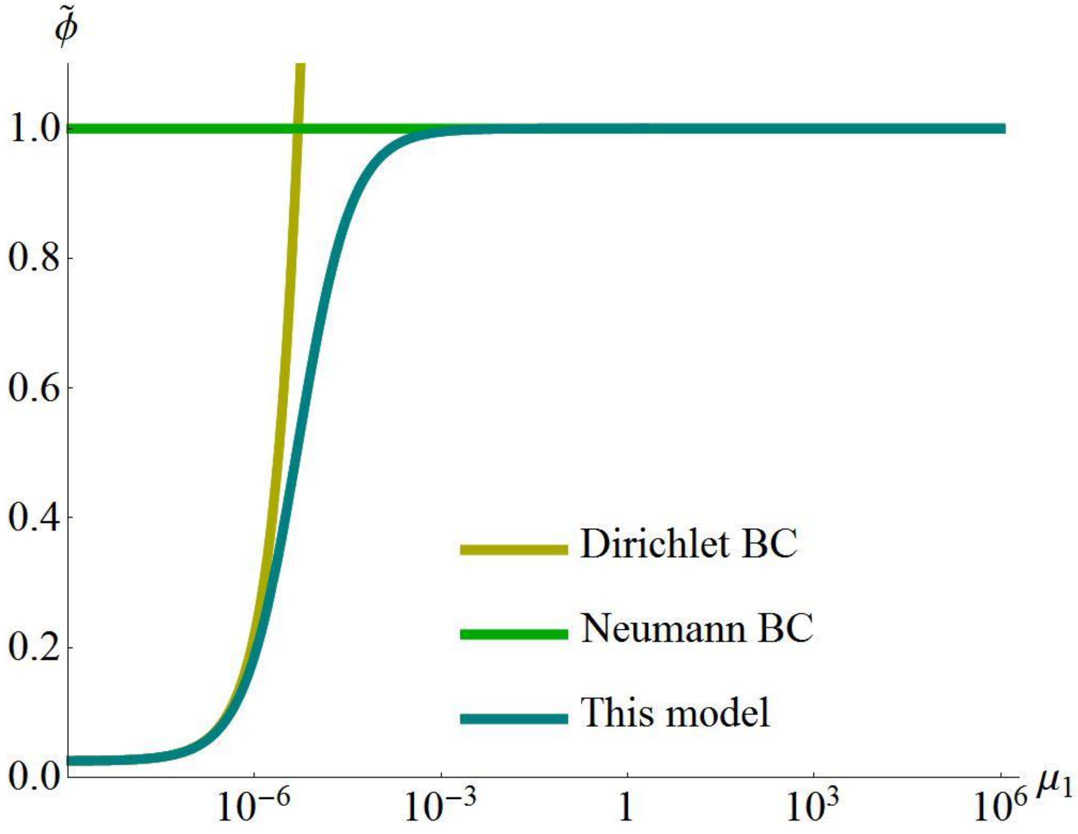
$$\mu_3 \equiv \theta_{\mathrm{TE}} d_{\mathrm{TE}} / (T_{\mathrm{A}} \kappa_{\mathrm{TE}})$$

$$\mu_4 \equiv \kappa_{\mathrm{H}} / (d_{\mathrm{H}} \bar{h}_{\mathrm{h}})$$

$$\mu_5 \equiv \kappa_{\mathrm{TE}} / (d_{\mathrm{TE}} \bar{h}_{\mathrm{c}})$$

$$\mu_6 \equiv \kappa_{\mathrm{TE}} / \kappa_{\mathrm{H}}$$

Reconciling with Neumann and Dirichlet



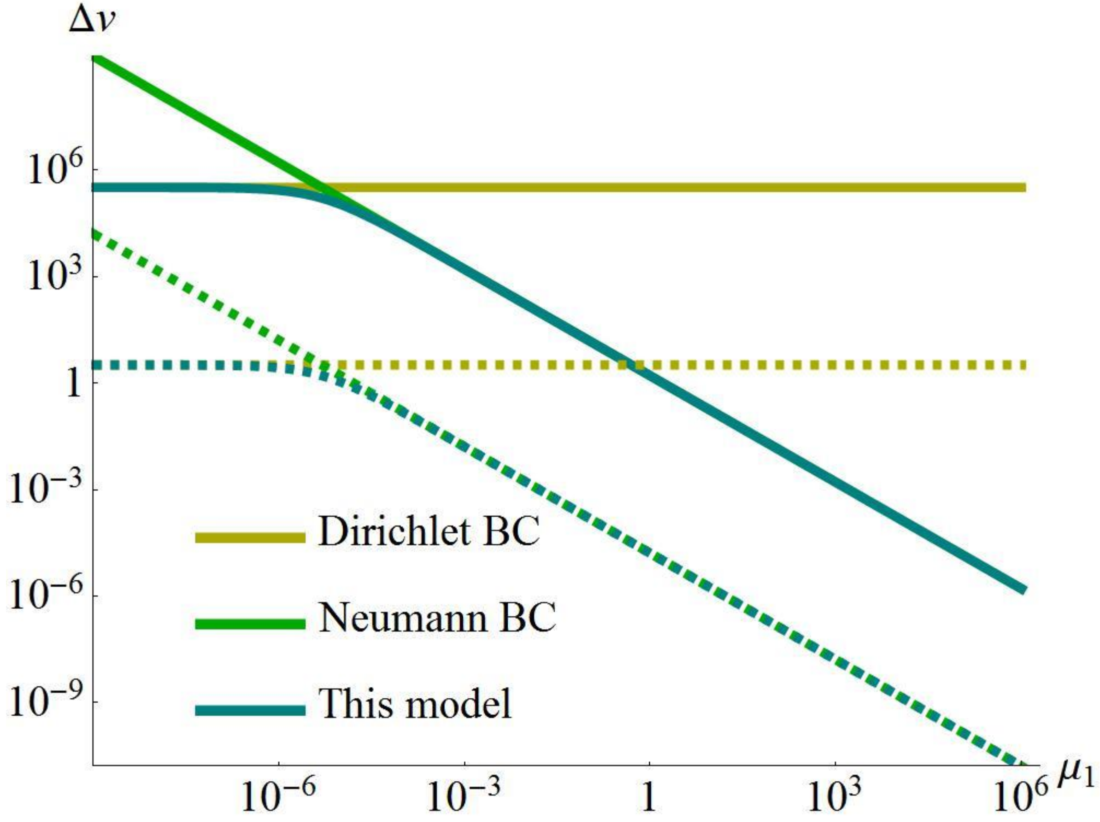
$$\mu_1 \equiv d_{\mathrm{H}} / d_{\mathrm{TE}}$$

$$\tilde{\phi} \equiv \phi / \theta_{\mathrm{H}}$$

For ideally dissipating cold sides and perfectly insulated heat sources ($1/\mu_4 = \mu_5 = 0$) constant heat flow BCs are recovered for $\mu_1 > 1$ (Neumann)

Reconciling with Neumann and Dirichlet

TEG



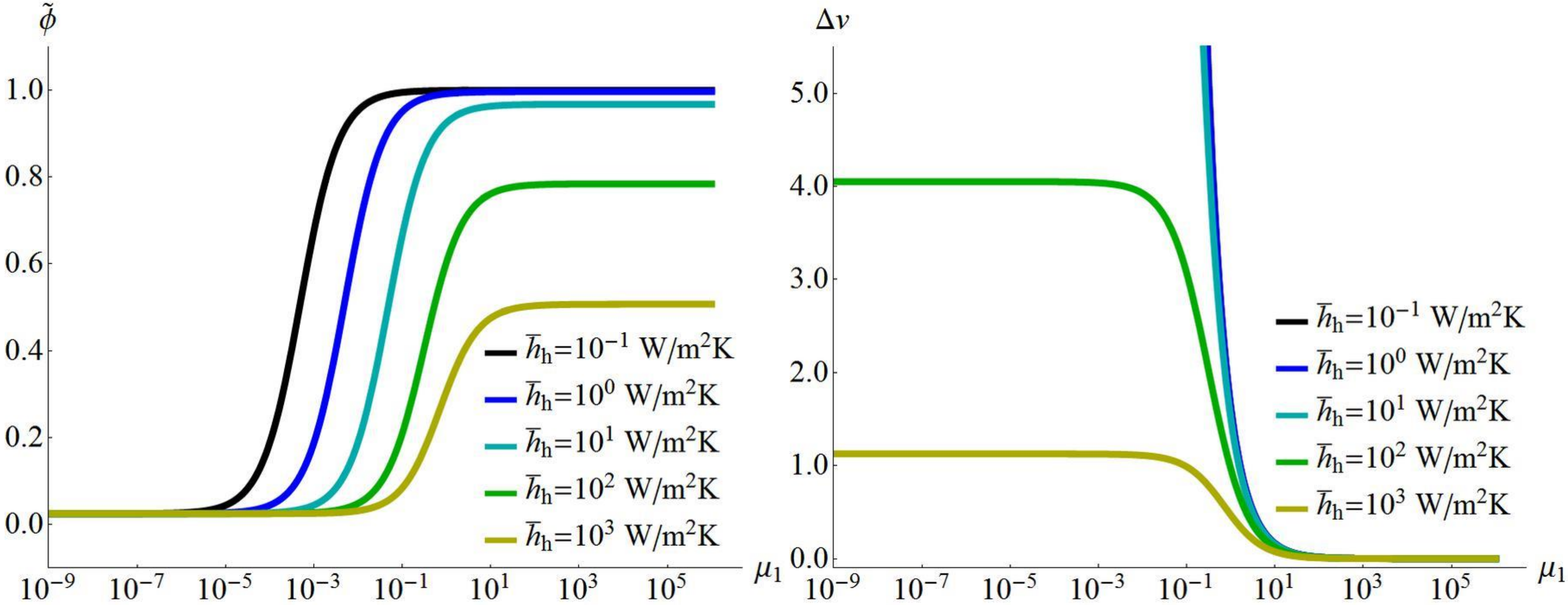
For ideally dissipating cold sides and perfectly insulated heat sources ($1/\mu_4 = \mu_5 = 0$) constant temperature BCs are recovered for $\mu_1<1$ (Dirichlet)

Ideal dissipation conditions show that proper BCs switch upon $d_{\mathrm{H}}/d_{\mathrm{TE}}$ ratio

$$\mu_1 \equiv d_{\mathrm{H}} / d_{\mathrm{TE}}$$

$$\Delta v \equiv v_H - v_C = (T_{\mathrm{H}} - T_{\mathrm{C}}) / T_{\mathrm{a}}$$

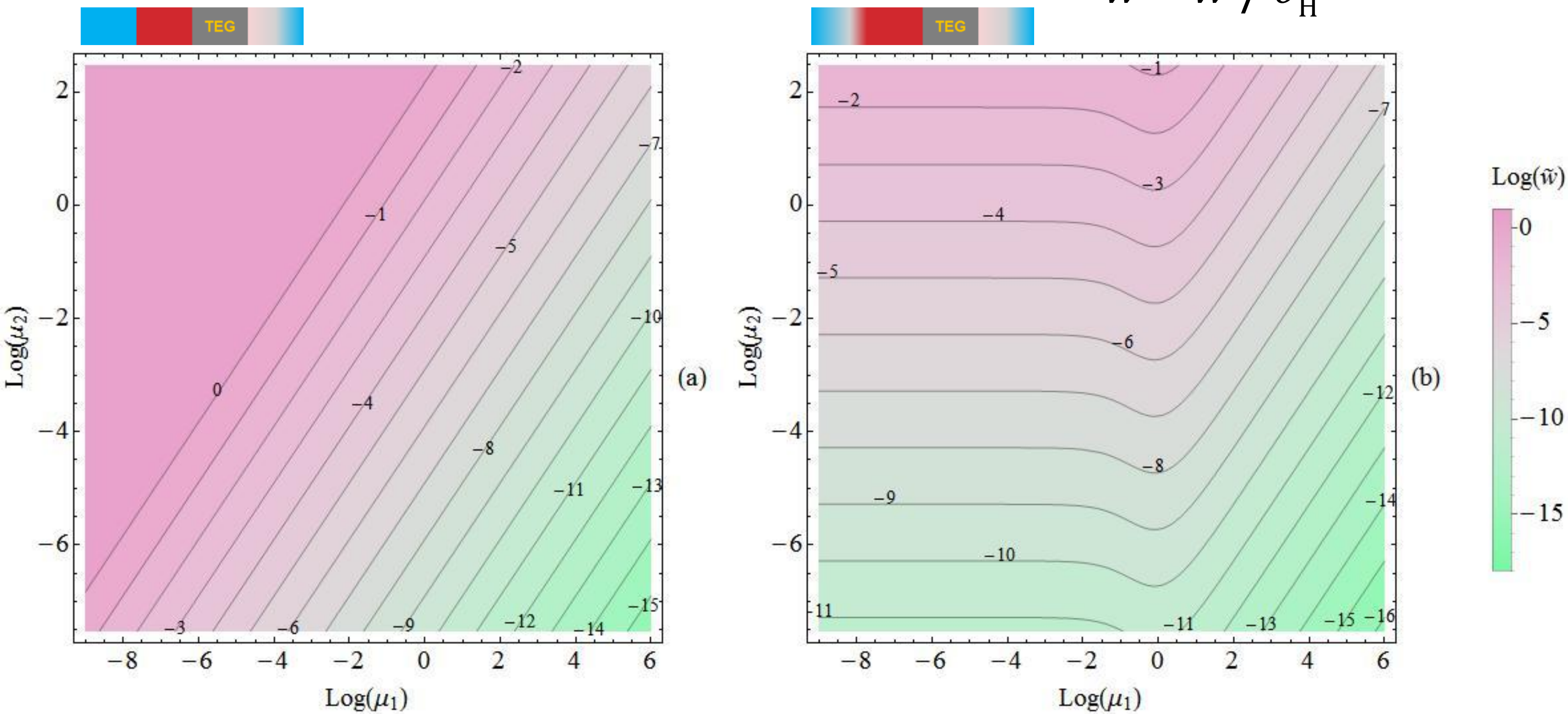
Dissipation efficiency



If the hot side is not perfectly insulated (or the heat source strength decreases) constant temperature/heat flow BCs do not apply around μ_1= 1. Since typical μ_1 for TEGs are between 10^{-1}- 10^{1} (bulk) and 10^{6} (micro/nano), application of standard analyses may mislead optimization of leg lengths.

Power output

$$\mu_2 \equiv \theta_H d_H / (T_a \kappa_H)$$
$$\tilde{w} \equiv w / \theta_H$$



For sub-ideal hot side insulation or low heat source strength, power output remains constant in the $\mu_1 \to 0$ limit. Thus, TE legs should fulfill $\frac{d_{TE}}{d_H} < \mu_1^*$(relevant for bulk TEGs).

ZT, PF, κ, and leg lengths

The solution of the ODE's reads

$$v_i(\hat{x}) = \sum_{j=0}^{2} (\beta_{ji} / \beta_{\mathrm{D}})\hat{x}^j \quad \text{with} \quad \beta_{ij} = \beta_{ij}(\ldots \mu_k \ldots)$$

$$\mu_1 \equiv d_{\mathrm{H}} / d_{\mathrm{TE}} \qquad \mu_2 \equiv \theta_{\mathrm{H}} d_{\mathrm{H}} / (T_{\mathrm{A}} \kappa_{\mathrm{H}}) = \theta_{\mathrm{H}} R_{\mathrm{H}} / T_{\mathrm{A}}$$

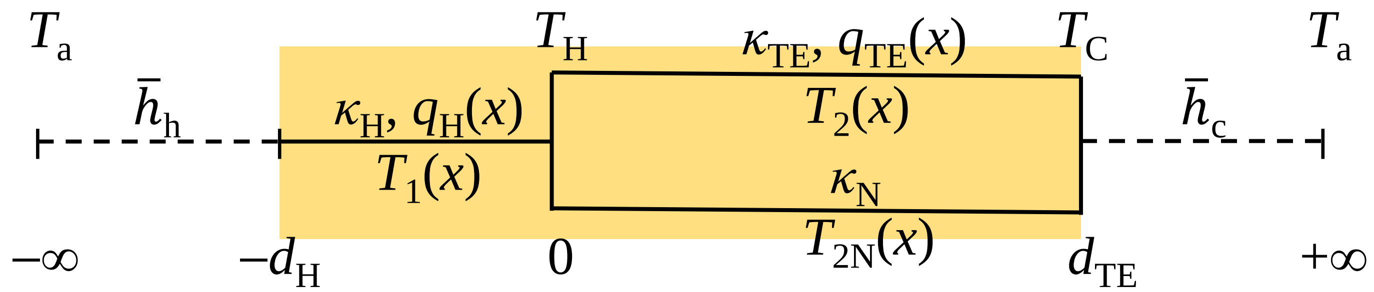
$$\mu_6 \equiv \kappa_{\mathrm{TE}} / \kappa_{\mathrm{H}} \qquad \mu_3 \equiv \theta_{\mathrm{TE}} d_{\mathrm{TE}} / (T_{\mathrm{A}} \kappa_{\mathrm{TE}}) = \theta_{\mathrm{TE}} R_{\mathrm{TE}} / T_{\mathrm{A}}$$

$$\mu_4 \equiv \kappa_{\mathrm{H}} / (d_{\mathrm{H}} \bar{h}_{\mathrm{h}}) = 1 / (R_{\mathrm{H}} \bar{h}_{\mathrm{h}}) \qquad \mu_5 \equiv \kappa_{\mathrm{TE}} / (d_{\mathrm{TE}} \bar{h}_{\mathrm{c}}) = 1 / (R_{\mathrm{TE}} \bar{h}_{\mathrm{c}})$$

Since κ's and d's enter the solution independently, the optimization of the TEG geometry will depend on material properties not only through ZT but also through PF and κ separately.

D. Narducci, J. Nanoeng. Nanomanuf. 1 (2011) 63–70.
D. Narducci, Appl. Phys. Lett. 99 (2011) 102104.

Branched (shunted) thermal circuits



$$
\begin{cases}
v_2''(\hat{x}) = \mu_3 \Pi(\hat{x} - ½) \\
v_2(1) = -\mu_5 v_2'(1) - \mu_5 \mu_7 \mu_6^{-1} v_{2N}'(1) \\
v_1''(\hat{x}) = -\mu_2 \mu_1^{-2} \Pi\left(\hat{x}\mu_1^{-1} + ½\right) \\
v_1(-\mu_1) = \mu_4 \mu_1 v'_1(-\mu_1) \\
v_1'(0) = \mu_6 v_2'(0) + \mu_7 v_{2N}'(0)
\end{cases}
$$

$$v_1(0) = v_2(0)$$

$$v_{2N}''(0) = 0$$

$$v_2(0) = v_{2N}(0)$$

$$v_2(1) = v_{2N}(1)$$

$$\mu_7 \equiv \kappa_N / \kappa_H$$

Impact on the design of micro/nanoharvesters

- Dirichlet and Neumann solutions recovered for ideally dissipating systems
- When dissipation is less than ideal, deviations from simplified models may be relevant both for micro and macro-harvesters depending on the characteristics of the heat source (power strength and insulation toward the ambient)
- From the material scientist viewpoint, once again ZT and κ should be thought as interdependent parameters.

Summary and Outlook

- A model allowing for a general analysis of TEGs with no need for a priori assumptions about BC has been presented
- Standard solution are recovered as limiting cases
- A transitional regime where neither fixed-temperature or fixed-heat flow BCs can be stipulated was found and modeled
- Predictions about optimal power outputs are found to depend on dissipation (well known) and on heat source strength and size (less obvious)

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